II European math tournament
Minsk, March 1–7, 2019

Senior Grand league. Math battle № 4. Battle for 5 and 7 places

March, 6

1. Non-zero real numbers x, y and z satisfy the equality $x + 2y + 4z = 0$. Find all the possible values of the expression $\dfrac{x^2}{8yz} + \dfrac{y^2}{zx} + \dfrac{8z^2}{xy}$.

2. There are fifty $+1$ and fifty -1 arranged in a circle so that the equal numbers are not neighboring. Each minute Vasya erases one of these numbers and writes down the sum of the erased number and the two neighbors. Prove that in 98 minutes the product of 98 numbers, that he wrote down, will be positive.

3. In a convex polygon with 1000 vertices no three diagonals have a common internal point. Some of 500 diagonals are red, and the rest are blue. From each vertex exactly one red diagonal comes out; each blue diagonal is intersected by at least one red diagonal. Find the minimal possible number of pairwise intersections of red diagonals.

4. A sequence of numbers 0 and 1 satisfy the following condition. For every $k > 2018$ $a_k = 0$ if $a_{k-2018} + \ldots + a_{k-1} > 23$ and $a_k = 0$ if $a_{k-2018} + \ldots + a_{k-1} \leqslant 23$. Prove that there exists such N that for all $n > N$ the equality $a_{n+2019} = a_n$ holds true.

5. There are one point marked on each side of a triangle, and there is a point P inside of the triangle. The point P is connected by segments with the vertices and the three marked points on the sides. Is it possible for the six formed triangles to be isosceles?

6. In how many ways can we paint the cells of a square $n \times n$ red or blue so that each square 2×2 has exactly two blue and exactly two red cells?

7. We are given a word with no more than distinct 10 letters (for example, a word PARALLELOGRAM). Prove that we can replace letters with digits (identical letters with identical digits, different letters with different digits) so that the resulting number would be divisible by 9.

8. There is a point C marked on a segment AB. Distinct points X and Y are such points that the triangles ABX and ACY are equilateral. Denote the midpoints of YC by K and the midpoint of BX by M correspondingly. It turns out that the triangle AMK is a right triangle. Find the quotient AC/BC.

International league. Math battle № 4. Sofia vs MMMF1329 (2)

March, 6

1. We are given triangles with integer angles (measured in degrees), and for each possible set of angles there is exactly one triangle. Let's call a triangle *good* if it can be split by a bisector into two smaller triangles with at least one of them being isosceles. How many good triangles are there?

2. Three kids have stored some chocolate. Each morning they weigh all chocolate. If one of those kids has at least 70% of chocolate the rest have, then he has to spare a half of his chocolate, and they all eat it together. At any other time nobody eats chocolate. Could it be possible that in a few days everyone has less chocolate than initially?

3. There are 70 kids sat at the round table. Could it be possible that exactly a half of them sit between a boy and a girl?

4. A checkered square 10×10 was cut along the sides of the cells into three polygons of the same perimeter. Find the greatest possible value of the perimeter.

5. There is a pile of 55 stones. Peter and Basil take turns, Peter starts. During the turn you have to split one of the piles into two lesser ones, and add two more piles that are equal to one of the piles that you just got (there is an infinite storage of extra stones). Peter wins if after his turn every pile has exactly one stone. Can Basil stop him?

6. Digits from 0 to 9 are written in the cells of a table 3×3, each digit is used no more than once. The sum of the numbers in the top row is four times greater than the sum in the bottom row. And the sum of the numbers in the right column is four times greater than the sum in the left column. Prove that the sum of the numbers in the middle row is equal to the sum of the numbers in the middle column.

7. Alex wrote down all the divisors of a positive integer N except 1 and N. After that he divided the maximal number he wrote by the minimal number and got 25 as the result. Find all the possible values of N.

8. There are 24 people in a club. Alice has 8 friends amongst the club members, and each of the rest have at least 11 friends. If someone from the club learns a rumour, he tells it to all of his friends. Alice learned a rumour. Prove that soon the whole club will definitely know it.

International league. Math battle № 4. Moscow-2Sch(2) vs Tajikistan (s)

March, 6

1. We are given triangles with integer angles (measured in degrees), and for each possible set of angles there is exactly one triangle. Let's call a triangle *good* if it can be split by a bisector into two smaller triangles with at least one of them being isosceles. How many good triangles are there?

2. Put the brackets in the left part of the equality $1 : 2 : 3 : 4 : 6 = 1$ so that it becomes correct. (It is not allowed to change the order of the numbers).

3. There are 20 kids sat at the round table. Could it be possible that exactly a half of them sit between a boy and a girl?

4. Cut a checkered square 5×5 along the sides of the cells into three parts of the same perimeter so that their areas differ no more than one cell.

5. There is a pile of 55 stones. Peter and Basil take turns, Peter starts. During the turn you have to split one of the piles into two lesser ones, and add two more piles that are equal to one of the piles that you just got (there is an infinite storage of extra stones). Peter wins if after his turn every pile has exactly one stone. Can Basil stop him?

6. Digits from 1 to 9 are written in the cells of a table 3×3, each digit is used no more than once. The sum of the numbers in the top row is four times greater than the sum in the bottom row. And the sum of the numbers in the right column is 1.5 times greater than the sum in the left column. Which digit could be in the centre of the table?

7. Alex wrote down all the divisors of a positive integer N except 1 and N. After that he divided the maximal number he wrote by the minimal number and got 25 as the result. Find all the possible values of N.

8. There are 24 people in a club. Alice has 8 friends amongst the club members, and each of the rest have at least 11 friends. If someone from the club learns a rumour, he tells it to all of his friends. Alice learned a rumour. Prove that soon the whole club will definitely know it.

II European math tournament
Minsk, March 1–7, 2019

International league. Math battle № 4. Ecuador vs Tajikistan

March, 6

1. The first digit of a 3 digit number is 5. This digit was moved to the last position. As the result the number decreased by 360. Find the original number.

2. Put the brackets in the left part of the equality $1 : 2 : 3 : 4 : 6 = 1$ so that it becomes correct. (It is not allowed to change the order of the numbers).

3. There are 20 kids sat at the round table. Could it be possible that exactly a half of them sit between a boy and a girl?

4. Cut a checkered square 5×5 along the sides of the cells into three parts of the same perimeter so that their areas differ no more than one cell.

5. Alex writes down positive integers in a row with no spaces: $123456789101112\ldots$ Find the numbers of positions where the combination of digits 555 appears for the first time.

6. Digits from 1 to 9 are written in the cells of a table 3×3, each digit is used no more than once. The sum of the numbers in the top row is four times greater than the sum in the bottom row. And the sum of the numbers in the right column is 1.5 times greater than the sum in the left column. Which digit could be in the centre of the table?

7. Alice and Bob each wrote a number. Vlad divided Alice's number by Bob's number and wrote down the result. It turned out that Alice's number is 7 times greater than Bob's number, and Bob's number is 5 times greater than Vlad's number. Find out what Alice's number is.

8. Three baseball players left one team and joined another team. Could it be possible that the average height in both teams has increased?