II European math tournament
Minsk, March 1–7, 2019

Senior Grand league. Math battle № 3

March, 5

1. In a convex quadrilateral $ABCD$ $\angle B = 50^\circ$, $\angle A = 80^\circ$. There is a point K marked on the extension of the side AB beyond the point A such that $AK = CD$. It turns out that $KD \perp BC$. Find out the value of the angle between AC and bisector of the angle BCD.

2. There are 2019 cities in a country. Between each pair of cities may be an airway provided by "Air fleet" or an airway provided by "Trouble" (but not both) or no airways at all. For any four of the cities these three conditions are satisfied:

1) The total number of airways between the cities is even;

2) There are two cities connected by an "Air fleet" airway';

3) If there are two cities connected by "Trouble", then the number of "Trouble" airways between these 4 cities is not less then "Air fleet" airways between them.

What the greatest number of the cities can be chosen in the country so there is an "Air fleet" airway between any two chosen cities, no matter how the cities connected?

3. Could there exist such 101 positive integers (not necessarily distinct) whose product is equal to the sum of their all pairwise LCMs (the least common multiples)?

4. p is a prime number, greater than 2. Find out all positive integer n such that the numbers $1, 2, 3 \ldots, n-1, n$ can be separated into p disjoint groups with equal sums.

5. There are n positive numbers x_1, x_2, $\ldots$, x_n. Peter wrote n fractions $\frac{x_1-x_2}{x_1+x_2}$, $\frac{x_2-x_3}{x_2+x_3}$, $\ldots$, $\frac{x_{n-1}-x_n}{x_{n-1}+x_n}$, $\frac{x_n-x_1}{x_n+x_1}$ on a board. Vasya wrote in his notebook all possible products of odd amount of fractions which was written by Peter . Prove that the sum of the numbers were written in the Vasya's notebook is equal to zero.

6. Let's call a pair of distinct 9-digit number n and m *amazing*, if m and n differ only in a permutation of digits and there aren't two identical digits in n, and n is divisible by m. Prove that each number in an amazing pair contains a digit 8. Don't forget that the first digit of each positive integer isn't 0.

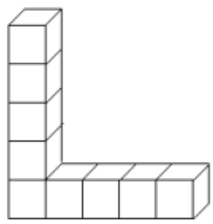
7. There are 50 athletes with different height and different age in a row. Exactly 20 of them are older than the neighbour on the right. Exactly 19 of them are higher than the neighbour at the left. Prove that there exists an athlete who is higher and older than the neighbour on the right.

8. Five clever children sit at a round table. The teacher loudly sad: "I gave several apples to each of you so that all children would have a different amount of apples. I distributed k apples." Also he quietly told each child how many apples his neighbors at the left and on the right have. Each child knows how many apples he has and wants to guess the difference between the amounts of apples of two children sitting on the opposite of him. Children call the numbers at the same time, without discussing anything beforehand. Define the maximum k so that there will be at least one child who is able to guess the difference.

International league. Math battle № 3

March, 5

1. We are given a figure made of 9 cubes of the same size (see pic.) Is it possible to cover its surface with 3 rectangles without gaps and overlapping? (Rectangles can be folded over the edges. Rectangles can be of different size.)



2. We are give a line with a point O on it, dividing the line into two rays. Alex draws two more rays starting from O, both rays are at the same side of the line, they build a 50 angle. Now Alex has 4 rays in total. For each of 6 pairs of rays he found what angle they build (one is 180, another is 50 and there are 4 more angles). Alex found that the sum of the maximal acute angle and the maximal obtuse angle is 200. Find the value of the maximal acute angle.

3. In a row of 50 sportsmen all have different height and different age. Exactly 20 sportsmen are older than their right neighbor. Exactly 19 sportsmen are taller than their leftt neighbor. Prove that there is a sportsmen who is both older and taller than his right neighbor.

4. In some kingdom only the coins of integer value in dinars are valid, and any value less or equal to 20 dinars can be paid with 3 or less coins. What is the least possible number of values in the kingdom?

5. Santa makes 12 rounds along the ice ring moving with a constant velocity. He and Snow White started at the same point at the same moment. Snow White is 3 times slower than Santa. During the moments when Santa increases his distance to Snow White she is pouring tears, otherwise she doesn't cry. Find out what part of the ring length will be covered in tears? (The distance between Santa and Snow White is measured along the ring).

6. Each of 33 cards has a digit on its face. All the cards lie on the table with its face down. The pupil knows that some card has 5 on its face. He turns over cards one by one. When he sees a digit he should insert plus or minus before the digit. Only then he is allowed to turn over the next card. How can the pupil put signs to make sure that the sum of all 33 numbers is not divisible by 6?

7. We call an integer bad if all its digits are divisible by 3 (e.g. integers 9, 36, 3093 are bad, while 10 and 3569 are not bad). From the row of integers from 1 to 1000 all bad integers are erased, and the rest were written in the increasing order and turned into the sum by putting signs − and + alternatively (here is the beginning of the sum: $-1 + 2 - 4 + 5 - 7 + 8 - 10 + 11 - 12 + 13 \ldots$) Calculate the sum.

8. 10 pupils are sat at the round table. Each one thought of a number and whispered it to his both neighbours. After that each pupil calculated the sum of two numbers whispered by his neighbours. Asked clockwise the pupils said these sums, namely $2, 4, 6, \ldots, 20$. Find out what number the pupil, who said aloud the sum 12, had thought of.

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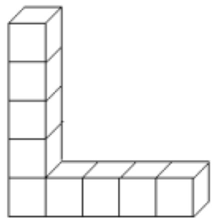

2. Strong man with each blow breaks the concrete slab or its part into 3 smaller parts. How many blowes will he need to break the slab into 81 pieces?

3. In a row of 50 sportsmen all have different height. Exactly 15 sportsmen are taller than their left neighbor. How many sportsmen are taller than their right neighbor?

4. Find the maximum seven-digit number divisible by 36.

5. Alice, Bob and Vlad each got the same pair of numbers. Alice added her own numbers, Bob multiplied, and Vlad subtracted one from the other. All the results occured to be the same. Find the original pair of numbers.

6. The store sells pencils of three colors, three different lengths and at three different prices. Is it always possible to choose from them three pencils so that they all have different colors, different lengths and different prices?

7. Each cell of a checkered square 20×20 was cut in both diagonals. Determine how many pieces there are. (For example, a square 2×2 cut in such a way breaks down into 12 parts).

8. 10 pupils are sat at the round table. Each one thought of a number and whispered it to his both neighbours. After that each pupil calculated the sum of two numbers whispered by his neighbours. Asked clockwise the pupils said these sums, namely 2, 4, 6, . . . , 20. Find out what number the pupil, who said aloud the sum 12, had thought of.